

Pulsed quantum interaction between two distant mechanical oscillators

Nikita Vostrosablin,^{1,*} Andrey A. Rakhubovsky,¹ and Radim Filip¹

¹*Department of Optics, Palacký University, 17. Listopadu 12, 771 46 Olomouc, Czech Republic*

Feasible setup for pulsed quantum non-demolition interaction between two distant mechanical oscillators through optical or microwave mediator is proposed. The proposal uses homodyne measurement of the mediator and feedforward control of the mechanical oscillators to reach the interaction. To verify quantum nature of the interaction, we investigate the Gaussian entanglement generated in the mechanical modes. We evaluate it under influence of mechanical bath and propagation loss for the mediator and propose ways to optimize the interaction. Finally, both currently available optomechanical and electromechanical platforms are numerically analyzed. The analysis shows that implementation is already feasible with current technology.

PACS numbers: 42.50.Wk, 42.50.Dv, 42.50.Ex

I. INTRODUCTION

Quantum optomechanics and electromechanics connecting light and microwaves with mechanical motion at the quantum level is an emerging field of quantum physics and technology [1–3]. Recently, Gaussian quantum entanglement between mechanical oscillator and microwave field [4], and nonclassical photon-phonon correlation of mechanical membrane and optical pulse [5] have been experimentally demonstrated. Both experiments used modern pulsed optomechanics [6–8]. They open new possibilities to experimentally connect other physical platforms with mechanical oscillator, like continuous-variable cold atom ensembles [9–11], and further many discrete systems like individual atoms [9, 12], superconducting qubits [13, 14], solid-state systems [15–18] and semiconductor systems [19, 20]. Together with these interesting and challenging interdisciplinary experiments, state-of-the-art of laboratory techniques could currently allow to let interact two mechanical oscillators mediated by light or microwave field. It is another interesting step forward, two similar mechanical oscillators coupled at quantum level have not been demonstrated yet. It can be very stimulating especially because the connection between two mechanical systems can physically connect quantum optomechanics to classical thermodynamics. If two similar quantum mechanical oscillators will be interfaced by the quantum version of the coupling typically used in classical mechanics, they can naturally generate entanglement. It is a simple witness that they were coupled quantum mechanically. Additionally, the mechanical-mechanical interaction can be quantum non-demolition type, which is required for basic continuous-variable quantum gate [21], useful for its specific features, for both gate-based [22] and cluster-state-based [23] quantum computing. Recently, the nonlocal optical QND gate was demonstrated [24] following the theoretical proposals in [25, 26]. Such the QND coupling was already broadly exploited between two atomic en-

sembles [27]. It is therefore much more important for the future to achieve such the well-defined quantum interaction of mechanical oscillators, not only the generation of entangled state of two mechanical systems.

Generation of entanglement between two mechanical systems have been already proposed in three different configurations. In the first type of proposed setups, two mechanical oscillators have been placed in a single optical cavity [28–36]. In this case, the continuous generation of steady-state entanglement appears because the mechanical oscillators interact with joint optical intra-cavity field. This configuration has been extensively used to discuss continuous-time quantum synchronization [37–39]. In the second kind of proposals, two entangled beams of light were used to entangle two mechanical systems without necessity to measure them [40–42]. In the third kind of proposed setups, two continuous-wave beams of light, leaving two continuously pumped optomechanical cavities, are jointly detected in Bell measurement and photocurrent is used to correct the mechanical oscillators [43–45]. These schemes can generate entanglement at a distance, however, it is very limited because of instabilities in the blue-detuned continuous-wave regime. Advanced time-continuous quantum measurement and control has been suggested to prepare mechanical entanglement [46]. Recently, theoretical investigation of optomechanical crystals has offered many other ways how to obtain mechanical entanglement [47]. Our goal is to propose currently feasible scheme with potential to use power of quantum optics tools to complement recent experimental test of coupled quantized mechanical oscillations of trapped ions [48].

In this paper, we propose currently feasible way to build basic *pulsed* quantum non-demolition (QND) interaction between two mechanical oscillators at a distance, connected by light or microwave field. The scheme is depicted in Fig. 1. Using homodyne detection of light or microwave field and feedforward control, means of both mechanical oscillators precisely follow the QND interaction. To generate significant entanglement of mechanical oscillators, coherent light is sufficient, and the entanglement can be very well estimated when intra-cavity field can be adiabatically eliminated. On the other hand, squeezed

* nikita.vostrosablin@upol.cz

light is advantageous to approach ideal QND interaction between two mechanical systems. Feasible squeezing of light is capable to enhance entangling power of the QND interaction. However, for larger optomechanical coupling strength and larger squeezing, non-adiabatic methods taking the intracavity field fully into account are required. Importantly, the non-adiabatic calculations predict a decrease of the entanglement power for larger squeezing. It is due to presence of the intra-cavity field and the squeezing has to be therefore optimized to get maximum of entangling power. We prove sufficient stability of the QND interaction under influence of mechanical bath and transmission loss between two separated cavities. Finally, we verified that it is feasible to build the mechanical QND interaction for both current optomechanical [49] and electromechanical [50] setups.

The paper is organized in the following way. We begin by mathematical definition of quantum-nondemolition interaction and principal description of the experimental setup. First, in Sec. III we carry out a simple principal analysis of the physics of the setup. To do so we start from a brief derivation of equations of motion for an optomechanical system in Sec. III A and solve those in Sec. III B ignoring for a while the decoherence and eliminating intracavity modes. We quantify the interaction between the mechanical modes analyzing the transfer of first moments of quadratures, and for a figure of merit of the strength of the interaction we employ the entanglement between the modes. We use logarithmic negativity [51] as a measure of entanglement. We show principal possibility of the protocol performance and derive the simplest conditions on the experimental parameters.

Second, in Sec. IV we perform a full numerical analysis of the system allowing for the imperfections. Those include impact of the intracavity modes that mediate the interaction between the travelling light pulse and the mechanical modes and the thermal bath causing decoherence of the latter. We as well investigate the impact of the optical loss between the cavities. We show that with currently available parameters the protocol can establish a QND interface between the two distant mechanical modes.

II. SETUP FOR PULSED QND INTERACTION BETWEEN MECHANICAL OSCILLATOR

In this paper, we propose a feasible way of implementation of quantum non-demolition (QND) interaction between mechanical modes of two distant optomechanical cavities. The QND interaction of two harmonic oscillators may be described by Hamiltonian $\mathcal{H}_{\text{int}} = \hbar g Q_1 Q_2$ with $Q_{1,2}$ being the position or momentum of the corresponding oscillator and g , interaction strength. After the interaction both the variables Q_1 and Q_2 remain unperturbed (not demolished) whereas the complementary ones to Q_1 (Q_2) become linearly displaced by a value proportional to gQ_2 (gQ_1).

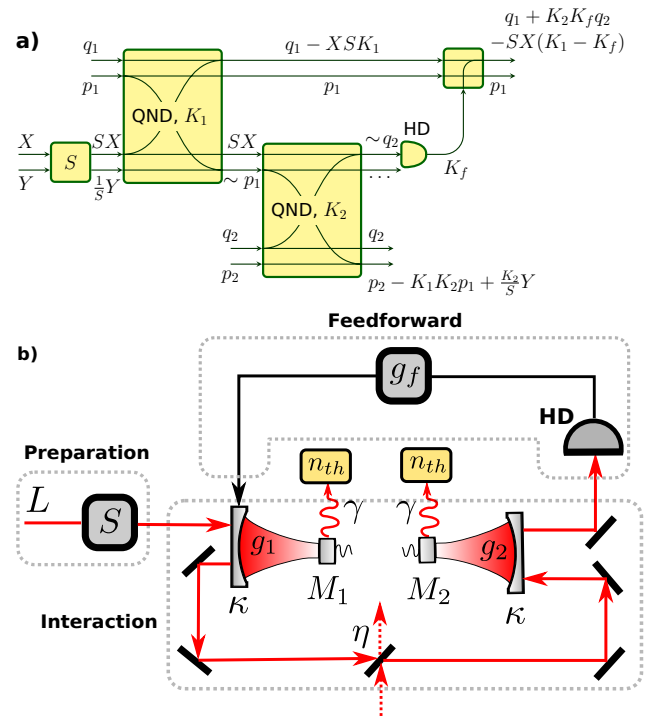


FIG. 1. The protocol of QND interaction between two mechanical modes. a) Simplified scheme, S — squeezing operation, HD — homodyne detector. b) possible experimental implementation with the imperfections: η — optical losses between the cavities, n_{th} — thermal mechanical environment.

The proposed scheme is presented in the Fig. 1. It is the simplest setup for generation of QND coupling between two mechanical systems. It is basically a serial scheme which does not require multiple pass of optical pulse through single optomechanical cavity. Moreover, it exploits advantage of squeezed light, homodyne detection, which are very efficient resources of quantum optics. The feedforward correction on mechanics can be done simply at any time by classical pulse of laser light. The modes of two mechanical oscillators M_1 and M_2 interact by turns with an optical (or microwave) pulse L via opto(electro-)mechanical coupling. The pulse is then detected and the result of the detection is used to linearly displace the mechanical mode of the first cavity (if needed, in the second one is displaced as well). The optical pulse can be prepared in a squeezed intensive coherent state and sent into the optomechanical cavity. The latter in essence comprises an optical mode coupled via radiation pressure to a mechanical harmonic oscillator [52]. We follow the standard approach [1, 53, 54] and assume that the optical pulse is displaced with a strong classical component that is modulated at mechanical frequency. This ensures that the effective interaction within the cavity is the non-demolition type.

The QND interaction allows a partial exchange of the variables between the mechanical mode M_1 and the travelling pulse (see Fig. 1 (a)). The latter is then redirected

to the second cavity with mechanical oscillator M_2 , which we assume to be identical to the M_1 . The QND interaction within the second cavity allows to transmit a variable of the mode M_1 carried by the pulse to the mode M_2 and in turn to transmit a variable of the mode M_2 to the light. The pulse is then detected and the result of detection is used to displace the mode M_1 to complete transfer of the M_2 variable. A proper strong presqueezing of the light pulse and the feed-forward correction allow to eliminate all variables from the final transformation of the mechanical modes that consequently approach an ideal QND interaction between them.

III. PERFORMANCE OF SETUP FOR QND INTERACTION

A. Optomechanical quantum non-demolition interaction

Let us first consider a single optomechanical cavity that in essence embodies an optical mode and a mechanical one. The two modes are coupled by radiation pressure with the Hamiltonian [52] $H_{\text{rp}} = -\hbar g_0 n_{\text{cav}} x/x_{\text{zfp}}$, where n_{cav} stands for intracavity photon number, x , mechanical displacement from equilibrium, g_0 , so-called single-photon coupling strength. The mechanical zero-point fluctuation amplitude, denoted by x_{zfp} , for a mechanical oscillator with mass m and eigen frequency ω_m equals $x_{\text{zfp}} = \sqrt{\hbar/2m\omega_m}$.

In order to enhance the radiation pressure coupling, strong coherent field is used as the pump. This allows to linearise the dynamics around a steady classical state and solve for quantum corrections. Moreover, we assume this strong classical field to be resonant with the cavity and modulated at the frequency of the mechanical oscillator [53]. In this case if the mechanical frequency ω_m exceeds all other characteristic frequencies of the system, one can perform averaging to get rid of the terms at $2\omega_m$ to obtain the non-demolition coupling. The latter condition is usually equivalent to the requirement that the optical decay rate κ of the cavity be smaller with respect to ω_m , known as resolved-sideband regime.

After the linearization and averaging out the rapid oscillating terms we arrive to the QND coupling within the optomechanical cavity with Hamiltonian that reads (depending on the phase of the pump)

$$\mathcal{H} = \hbar g X p \quad \text{or} \quad \mathcal{H} = \hbar g Y q, \quad (1)$$

where $g = g_0 \sqrt{\langle n_{\text{cav}} \rangle}$ is the enhanced optomechanical coupling strength, X and Y , and q and p are quadratures of, respectively, the optical and mechanical modes which obey usual commutation relations ($[X, Y] = i$; $[q, p] = i$). The mechanical displacement x can be expressed in terms of quadratures as $x/x_{\text{zfp}} = q \cos \omega_m t + p \sin \omega_m t$ and a similar expression holds for the optical quadratures.

To describe the interaction of the propagating light pulse with the optomechanical cavity we complement

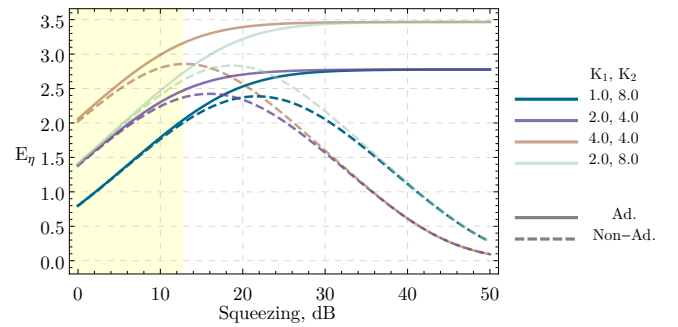


FIG. 2. Entanglement between the two mechanical modes as a function of optical presqueezing. Solid lines correspond to adiabatic solution; dashed lines to solution with cavity mode. Different colors are used for different ratio of the gains K_1 and K_2 . Losses are absent: $\eta = 1$. Highlighted is the region of squeezing magnitudes not exceeding the value of 12.7 dB reported in Ref. [56].

the Hamiltonian of the optomechanical interaction $\mathcal{H}_1 = -\hbar g_1 X_1 p_1$ with input-output relations [55] (henceforth we denote with index “1” or “2” quantities corresponding to the respective cavity). The system is thus described by the following set of equations:

$$\begin{aligned} \dot{q}_1 &= -\frac{\gamma}{2} q_1 - g_1 X_1 + \xi_{q1}, & \dot{X}_1 &= -\kappa X_1 + \sqrt{2\kappa} X^{\text{in}}, \\ \dot{p}_1 &= -\frac{\gamma}{2} p_1 + \xi_{p1}, & \dot{Y}_1 &= -\kappa Y_1 + \sqrt{2\kappa} Y^{\text{in}} + g_1 p_1 \\ Q^{\text{out}} &= \sqrt{2\kappa} Q - Q^{\text{in}}, & Q &= X, Y. \end{aligned} \quad (2)$$

Here $X^{\text{in}}, Y^{\text{in}}$ are the quadratures of the pulse with commutator $[X^{\text{in}}, Y^{\text{in}}(t')] = i\delta(t - t')$, $\xi_{q,p}$ are the quadratures of mechanical noise. κ and γ are respectively optical and viscous mechanical damping coefficients.

B. Adiabatic regime

As a first approximation we consider the system in adiabatic regime. Given that optical decay rate exceeds the other rates in (2) (which is typically the case in experiment), one can assume that the optical mode reacts to any changes instantaneously, which is equivalent to putting $\dot{X} = \dot{Y} = 0$ in Eqs (2). Formally this corresponds to replacement of all the functions of time with their own versions averaged over the interval with duration τ_* such that $\kappa \gg 1/\tau_* \gg \gamma, g$.

Lastly, in this section we leave out the mechanical decoherence, setting $\gamma = 0$, $\xi_{q1} = \xi_{p1} = 0$.

With these assumptions the solution of Eqs. (2) reads

$$\begin{aligned} q_1(\tau) &= q_1(0) - S K_1 \mathcal{X}^{\text{in}}, & \mathcal{X}_1^{\text{out}} &= S \mathcal{X}^{\text{in}}, \\ p_1(\tau) &= p_1(0), & \mathcal{Y}_1^{\text{out}} &= \frac{1}{S} \mathcal{Y}^{\text{in}} + K_1 p_1(0). \end{aligned}$$

We have introduced the squeezing magnitude S and the effective interaction strength $K_1 = g_1 \sqrt{2\tau/\kappa}$. We also

have defined the input and output quadratures of the cavity as

$$\mathcal{Q}^k = \frac{1}{\sqrt{\tau}} \int_0^\tau Q^k(s) ds, \quad Q = X, Y, \quad k = \text{in, out.}$$

The quadratures are normalized to obey $[\mathcal{X}^k, \mathcal{Y}^k] = i$.

The output field from the first cavity is then delivered to the input of the second one through a purely lossy channel that performs an admixture of vacuum to the signal, therefore

$$Q_2^{\text{in}} = \sqrt{\eta} Q_1^{\text{out}} + \sqrt{1-\eta} Q_{\text{ls}}, \quad Q = X, Y.$$

Here Q_{ls} are the quadratures of vacuum mode.

The optomechanical interaction within the second cavity is described by the Hamiltonian $\mathcal{H}_2 = \hbar g_2 Y_2 q_2$ and starts at time $t = \tau$. One can obtain the input-output relations for the second cavity in a similar fashion. For simplicity we assume the parameters of the second cavity (except the coupling g_2) to replicate the parameters of the first one.

The optical output quadrature $\mathcal{X}_2^{\text{out}}$ is measured and the position of the mechanical mode of the first cavity is displaced so that the final value equals $q_1 = q_1(\tau) + K_f \mathcal{X}_2^{\text{out}}$.

$$\begin{aligned} q_1 &= q_1(0) + K_2 K_f q_2(\tau) \\ &\quad - S \mathcal{X}^{\text{in}} (K_1 - K_f \sqrt{\eta}) + \mathcal{X}_{\text{ls}} K_f \sqrt{1-\eta}, \\ p_1 &= p_1(0), \\ q_2 &= q_2(\tau), \\ p_2 &= p_2(\tau) - K_1 K_2 p_1(0) \sqrt{\eta} \\ &\quad - \mathcal{Y}^{\text{in}} \frac{K_2 \sqrt{\eta}}{S} - K_2 \sqrt{1-\eta} \mathcal{Y}_{\text{ls}}. \end{aligned} \quad (3)$$

Similarly, we have introduced $K_2 = g_2 \sqrt{\frac{2\tau}{\kappa}}$ here.

To approach the ideal QND interaction of the two mechanical modes with Hamiltonian $\mathcal{H}_{\text{QND}} = \hbar K_1 K_2 \tau^{-1} p_1 q_2$ one needs to fulfill a few conditions. First, ensure low loss ($\eta \rightarrow 1$) to get rid of the noisy mode Q_{ls} . Second, pick a proper feed-forward gain $K_f = K_1 / \sqrt{\eta}$ and provide high squeezing $S \gg 1$ to suppress the optical mode \mathcal{Q}^{in} .

To quantify the strength of the interaction we estimate the entanglement between the two mechanical modes,

namely the logarithmic negativity [51] (see Appendix for details).

In the lossless case the optimal value of squeezing yielding maximum of entanglement is given by $S = |K_2 / (K_1 - K_f)|$. Therefore for the feedforward $K_f = K_1$ the entanglement increases with squeezing infinitely. In the limit of moderately strong coupling ($K_{1,2} \gtrsim 1$) the following approximation holds:

$$E_\eta \approx -\ln \frac{1}{2K_1 K_2} \sqrt{1 + \frac{K_2^2}{S^2}}. \quad (4)$$

From this expression follows that although increase of both S and $K_{1,2}$ leads to stronger entanglement, it is more efficient to increase K_1 . This can be seen from the latter equation in (3), where the noisy mediator quadrature \mathcal{Y}^{in} enters with a multiplier $\propto K_2$.

The LN for this simple model is presented as a function of the presqueezing S in Fig. 2 (solid lines). The parameters used for simulation are $\kappa/2\pi = 221.5$ MHz, $\gamma/2\pi = 328$ Hz, $\tau = 4.5$ μ s that correspond to a recent optomechanical experiment [49] with increased pulse duration τ .

From the Fig. 2 is clear that for low squeezing the LN is mostly defined by the interaction strength K_1 in the first cavity as it follows from (4). In the limit of high squeezing the LN saturates to the value that is defined by the product of gains $K_1 K_2$, again in agreement with (4).

IV. ROBUSTNESS TO IMPERFECTIONS

There are two sources of hindrance that we left out for the previous section. First is the intracavity modes that mediate interaction between the propagating pulse and the mechanical modes of interest. As well the intracavity modes produce unwanted memory effects that disturb the desired QND interaction. Second is the interaction of mechanical modes with the thermal environment.

In this section we first study these two sources independently and finally provide a full solution taking both into account simultaneously.

A. Impact of the intracavity modes

To consider the effect of the intracavity modes on the QND interface, we solve the set of dynamical equations (2) without the mechanical decoherence ($\gamma = 0$, $\xi_{q,p} = 0$). The solution reads (for compactness we write the solution for the lossless case, $\eta = 1$)

$$\begin{aligned}
q_1 &= q_1(0) + q_2(\tau)K_2K_f \left(1 - \frac{1 - e^{-\kappa\tau}}{\kappa\tau}\right) - S(K_1 - K_f) \frac{1}{\sqrt{\tau}} \int_0^\tau X_1^{\text{in}}(s) ds \\
&\quad + SK_1 \int_0^\tau X_1^{\text{in}}(s) \left(e^{-\kappa(\tau-s)} \left[1 - 4\kappa(\tau-s) \frac{K_f}{K_1}\right] \right) ds + X_1(0) \left(\frac{2g_f}{\kappa} [(1 - e^{-\kappa\tau}) - 2\kappa\tau e^{-\kappa\tau}] - \frac{g_1}{\kappa} [1 - e^{-\kappa\tau}] \right) \\
&\quad + X_2(0) \frac{2g_f}{\kappa} (1 - e^{-\kappa\tau}), \\
p_1 &= p_1(0), \\
q_2 &= q_2(\tau), \\
p_2 &= p_2(\tau) - p_1(0)K_1K_2 \left(1 + e^{-\kappa\tau} - \frac{2}{\kappa\tau}(1 - e^{-\kappa\tau})\right) - \frac{K_2}{S} \frac{1}{\sqrt{\tau}} \int_0^\tau \left(1 - e^{-\kappa(\tau-s)} [2\kappa(\tau-s) + 1]\right) Y_1^{\text{in}}(s) ds \\
&\quad - Y_1(0) \frac{2g_2}{\kappa} (1 - e^{-\kappa\tau}(1 + \kappa\tau)) - Y_2(0) \frac{g_2}{\kappa} (1 - e^{-\kappa\tau}),
\end{aligned} \tag{5}$$

where we defined $g_f \equiv K_f \sqrt{\kappa/2\tau}$.

These equations deviate from the idealized set (3) by presence of the initial intracavity quadratures $Q_{1,2}(0)$. As well the pulse quadratures Q^{in} can no longer be eliminated completely by a proper choice of K_f and high squeezing S . Moreover, in this case high squeezing amplifies the noisy summand with X^{in} degrading the interface.

From the Eqs. (5) follows that in the limit $\kappa \gg g_{1,2,f}$ and $\kappa\tau \gg 1$ these equations reduce to the pure QND transformations (3). Furthermore, from the first equation it follows that the effect of the unwanted summand $\propto SX^{\text{in}}$ can be reduced by decreasing K_1 . This is illustrated in Fig. 2 where we plot the LN for solution including the cavity modes as a function of squeezing for different couplings. At high squeezing the full solution deviates from the adiabatic one, however the curves with lower K_1 show this deviation at higher squeezing than the curves with higher K_1 .

The proper choice of the coupling thus allows to approach the performance of the idealized adiabatic regime. Note that in order to increase the LN it is more efficient to increase K_1 than K_2 . To increase the LN staying close to the preferred adiabatic regime (and therefore a pure QND interface between the mechanical modes) on the contrary it is preferable to increase K_2 .

B. Mechanical thermal bath

Finally we consider the system in presence of the thermal mechanical environment.

We assume that each of the mechanical modes is coupled at rate γ to its own environment that is in a thermal state with occupation n_{th} (see Fig. 1). The coupling for both modes takes place during the interaction with the pulse. Moreover, the first mode remains coupled to the environment during the interaction of the second system with the pulse. Before the interaction with the pulse the mechanical modes are in the ground state (the possibility

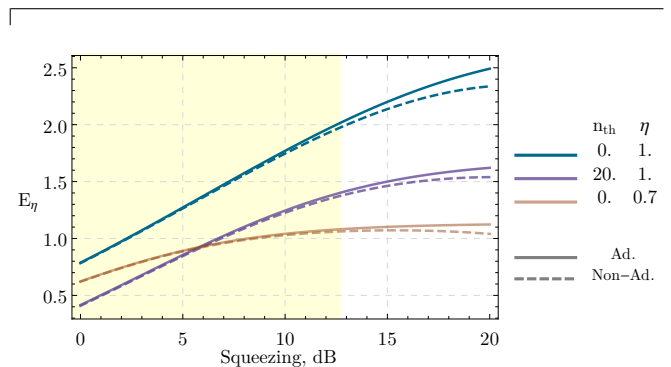


FIG. 3. Entanglement as a function of squeezing in presence of mechanical bath with mean number of phonons n_{th} and optical losses with transmittivity η . The optomechanical gains equal $K_1 = 1$, $K_2 = 8$, same as for the blue line in Fig. 2

to precool mechanical oscillator close to the ground state has been demonstrated for a number of setups [5, 57, 58]).

The thermal bath is represented in the equations (2) by Langevin force quadratures $\xi_{q,p}$. These quadratures are assumed Markovian so that

$$\begin{aligned}
\langle \xi_a(t)\xi_a(t') + \xi_a(t')\xi_a(t) \rangle &= \gamma(2n_{\text{th}} + 1)\delta(t - t'), \quad a = q, p; \\
\langle \xi_q(t)\xi_p(t') + \xi_p(t')\xi_q(t) \rangle &= 0.
\end{aligned}$$

The LN in adiabatic regime with intracavity modes eliminated is approximately given by (here $K_1 = K_2 = K$)

$$E_\eta \approx -\ln \frac{1}{2K^2} \sqrt{1 + \Gamma K^4 + \frac{K^2}{S^2}(1 + \Gamma K^4)}, \quad \Gamma = 2\gamma\tau n_{\text{th}}. \tag{6}$$

In case of zero mechanical damping the expression is reduced to (4).

The LN corresponding to the full solution with all the imperfections is plotted as a function of the squeezing S in Fig. 3 for a set of different parameters.

The main means how the mechanical environment affects the entanglement is adding the thermal noise to the

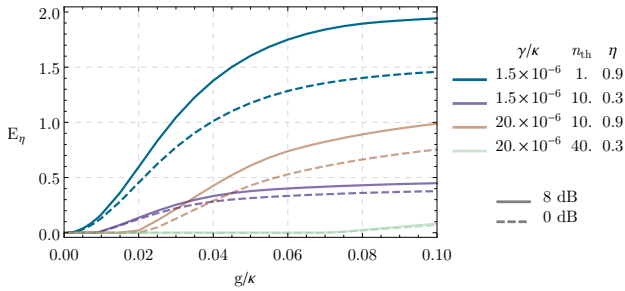


FIG. 4. Maximal entanglement achievable with the coupling rate g [in units of κ], for optomechanical parameters [49] (blue and violet lines) and electromechanical [50] (brown and green).

mechanical quadratures. Besides this the environment also creates small imbalance that prohibits the perfect cancellation of the optical mediator mode in q_1 by feedforward. The magnitude of this imbalance is however almost negligible.

We as well plot the LN as a function of the squeezing for nonzero loss ($1 - \eta \neq 0$). The Fig. 3 shows that at higher squeezing the entanglement between the mechanical modes is more tolerant to the mechanical bath than to the optical loss. Nevertheless, even with realistic loss parameters the entanglement does not vanish. We observe that adiabatic elimination is capable to very well fit the results for wide range of feasible squeezing of radiation.

Numerical analysis shows that the nonzero occupation of the mechanical bath creates a threshold for the coupling that allows the entanglement. At the same time, nonzero optical loss only decreases the value of the LN, so in case of zero occupation of the bath, the entanglement can tolerate any finite loss.

C. Coupling optimization for experiments [49, 50]

In prior sections we focused on approaching a pure QND interaction between the two mechanical modes. Therefore we assumed the feedforward to be adjusted in a way that helps to cancel most of the optical mediator quadrature X^{in} , i.e. $K_f = K_1/\sqrt{\eta}$. Now we aim for maximization of the entanglement between the two modes. We waive the constraint on K_f and numerically optimize the logarithmic negativity with respect to the optomechanical gains $K_{1,2}$, feedforward strength K_f , and the pulse duration τ given a limitation on the coupling strength.

The results of the numerical optimization are presented in Fig. 4. The optimal regime to achieve maximal entanglement appears to be very close to the regime of pure QND between the mechanical oscillators with long pulses $\kappa\tau \gg 1$ and $K_f = K_1/\sqrt{\eta}$.

Squeezed source of radiation apparently helps to improve entanglement in both opto- and electromechanical scheme for large η close to perfect transmission and

smaller n_{th} . Simultaneously, the threshold for g/κ to observe entanglement is lowered as well for higher η and lower n_{th} . On the other hand, for larger n_{th} and lower η , the squeezing of radiation is not important, however, we can still observe entanglement of mechanical systems if γ/κ is not too large and g/κ is sufficiently large. It is therefore fully feasible to generate entanglement with state-of-the-art systems.

The optomechanical setup noticeably outperforms the electromechanical one due to higher eigenfrequency of the mechanical oscillator and consequently lower bath occupation. The high occupation of the mechanical thermal bath in the electromechanical setup places a constraint on the available pulse durations which in turn limits the QND gain K .

V. CONCLUSION

We have proposed feasible way of the simplest pulsed implementation of entangling quantum non-demolition coupling between two distant but very similar mechanical oscillators, implementable with both current electromechanical and optomechanical setups. The method exploits squeezed light and microwave radiation and highly efficient homodyne detection to induce maximal entanglement for this purely mechanical coupling. We verified robustness of the procedure under small transmission loss between the oscillators and under mechanical thermal baths. We realized that both current optomechanical [49] and electromechanical [50] setups are sufficient for the implementation of an extended version of multiple QND interaction. It will allow pulsed studies of quantum synchronization of mechanical objects [37–39]. Afterwards, a detailed study of quantum interaction of possibly very different mechanical systems is important for development of physical connection with quantum thermodynamics [59–62]. The method can be further extended to controllably couple more mechanical systems in future by different type of Gaussian interactions and possibly challenging non-Gaussian transformations.

ACKNOWLEDGMENTS

We acknowledge Project No. GB14-36681G of the Czech Science Foundation. A.A.R. acknowledges support by the Development Project of Faculty of Science, Palacky University. N.V. acknowledges the support of Palacky University (IGA-PrF-2016-005).

Appendix: Logarithmic negativity

The mechanical modes in our system are initially in thermal states and the optical modes are all in vacuum, and the linear dynamic preserves the Gaussianity of the

states of mechanical modes. A Gaussian state of a two-mode system with quadratures $f = [q_1, p_1, q_2, p_2]^T$ is fully determined by a vector of means $\langle f \rangle$ and a covariance matrix (CM) with elements defined as

$$V_{ij} = \frac{1}{2} \langle \Delta f_i \Delta f_j + \Delta f_j \Delta f_i \rangle.$$

Here angular brackets denote the averaging over the quantum state, and $\Delta f_i \equiv f_i - \langle f_i \rangle$.

Covariance matrix may be divided into 2×2 blocks such that:

$$V = \begin{bmatrix} \mathcal{V}_1 & \mathcal{V}_c \\ \mathcal{V}_c^T & \mathcal{V}_2 \end{bmatrix},$$

where \mathcal{V}_1 and \mathcal{V}_2 characterize internal correlations in me-

chanical subsystems. The matrix \mathcal{V}_c stands for the correlations between the first and second mechanical modes. The diagonalisation of the CM leads to symplectic eigenvalues ν_{\pm} [51]:

$$\nu_{\pm} = \sqrt{\frac{1}{2} \left(\Sigma(V) \pm \sqrt{\Sigma(V)^2 - 4 \det V} \right)},$$

with

$$\Sigma(V) = \det \mathcal{V}_1 + \det \mathcal{V}_2 - 2 \det \mathcal{V}_c.$$

Logarithmic negativity is defined then as $E_{\eta} = \max[0, -\ln 2\nu_-]$ and we use it as the measure of the entanglement of the system under the consideration.

-
- [1] Markus Aspelmeyer, Tobias J Kippenberg, and Florian Marquardt, "Cavity optomechanics," *Reviews of Modern Physics* **86**, 1391–1452 (2014).
- [2] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity Optomechanics* (Springer Verlag Berlin Heidelberg, 2014).
- [3] W.P. Bowen and Milburn G.J., *Quantum Optomechanics* (Taylor&Francis Group, 2015).
- [4] T.A. Palomaki, J.D. Teufel, R.W. Simmonds, and K.W. Lehnert, "Entangling mechanical motion with microwave fields," *Science* **342**, 710–713 (2013).
- [5] Ralf Riedinger, Sungkun Hong, Richard A. Norte, Joshua A. Slater, Juying Shang, Alexander G. Krause, Vikas Anant, Markus Aspelmeyer, and Simon Gröblacher, "Non-classical correlations between single photons and phonons from a mechanical oscillator," *Nature* **530**, 313–316 (2016), arXiv: 1512.05360.
- [6] Sebastian G. Hofer, Witlief Wicczorek, Markus Aspelmeyer, and Klemens Hammerer, "Quantum entanglement and teleportation in pulsed cavity optomechanics," *Physical Review A* **84**, 052327 (2011).
- [7] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Ć Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, "Pulsed quantum optomechanics," *Proceedings of the National Academy of Sciences* **108**, 16182–16187 (2011), arXiv:1011.0879 [cond-mat, physics:quant-ph].
- [8] M. R. Vanner, J. Hofer, G. D. Cole, and M. Aspelmeyer, "Cooling-by-measurement and mechanical state tomography via pulsed optomechanics," *Nature Communications* **4**, 2295 (2013), arXiv: 1211.7036.
- [9] K. Hammerer, M. Aspelmeyer, E. S. Polzik, and P. Zoller, "Establishing Einstein-Poldosky-Rosen Channels between Nanomechanics and Atomic Ensembles," *Physical Review Letters* **102**, 020501 (2009).
- [10] S. Camerer, M. Korppi, A. Jockel, D. Hunger, T. W. Hansch, and P. Treutlein, "Realization of an optomechanical interface between ultracold atoms and a membrane," *Physical Review Letters* **107**, 223001 (2011).
- [11] A. Jockel, A. Faber, T. Kampschulte, M. Korppi, M. T. Rakher, and P. Treutlein, "Sympathetic cooling of a membrane oscillator in a hybrid mechanical-atomic system," *Nature nanotechnology* **10**, 55–59 (2015).
- [12] B. Vogell, T. Kampschulte, M. T. Rakher, A. Faber, P. Treutlein, K. Hammerer, and P. Zoller, "Long distance coupling of a quantum mechanical oscillator to the internal states of an atomic ensemble," *New Journal of Physics* **17**, 043044 (2015).
- [13] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, , and A. N. Cleland, "Quantum ground state and single-phonon control of a mechanical resonator," *Nature* **464**, 697 (2010).
- [14] J. M. Pirkkalainen, S. U. Cho, J. Li, G. S. Paraoanu, P. J. Hakonen, and M. A. Sillanpaa, "Hybrid circuit cavity quantum electrodynamics with a micromechanical resonator," *Nature* **494**, 211 (2013).
- [15] O. Arcizet, V. Jacques, A. Siria, P. Poncharal, P. Vincent, and S. Siedelin, "A single nitrogen-vacancy defect coupled to a nanomechanical oscillator," *Nature Physics* **7**, 879 (2011).
- [16] Shimon Kolkowitz, Ania C. Bleszynski Jayich, Quirin P. Unterreithmeier, Steven D. Bennett, Peter Rabl, J. G. E. Harris, and Mikhail D. Lukin, "Coherent Sensing of a Mechanical Resonator with a Single-Spin Qubit," *Science* **335**, 1603–1606 (2012).
- [17] J. Teissier, A. Barfuss, P. Appel, E. Neu, and P. Maletinsky, "Strain coupling of a nitrogen-vacancy center spin to a diamond mechanical oscillator," *Physical Review Letters* **113**, 020503 (2014).
- [18] Preeti Ovartchaiyapong, Kenneth W. Lee, Bryan A. Myers, and Ania C. Bleszynski Jayich, "Dynamic strain-mediated coupling of a single diamond spin to a mechanical resonator," *Nature Communications* **5**, 4429 (2014), arXiv: 1403.4173.
- [19] I. Yeo, P.-L. de Assis, A. Gloppe, E. Dupont-Ferrier, P. Verlot, N. S. Malik, E. Dupuy, J. Claudon, J.-M. Gerard, A. Auffeves, G. Nogues, S. Seidelin, J.-P. Poizat, O. Arcizet, and M. Richard, "Strain-mediated coupling in a quantum dot-mechanical oscillator hybrid system," *Nature nanotechnology* **9**, 106 (2014).
- [20] M. Montinaro, G. Wust, M. Munsch, Y. Fontana, E. Russo-Averchi, M. Heiss, A. Fontcuberta i Morral, R. J. Warburton, and M. Poggio, "Quantum dot optomechanics in a fully self-assembled nanowire," *Nano Lett.*

- 14, 4454 (2014).
- [21] S.D. Bartlett, B.C. Sanders, S.L. Braunstein, and K. Nemoto, “Efficient classical simulation of continuous variable quantum information processes,” *Physical Review Letters* **88**, 097904 (2002).
- [22] J. Yoshikawa, Y. Miwa, A. Huck, U.L. Andersen, P. van Loock, and A. Furusawa, “Demonstration of a quantum nondemolition sum gate,” *Physical Review Letters* **101**, 250501 (2008).
- [23] Y. Miwa, R. Ukai, J. Yoshikawa, R. Filip, P. van Loock, and A. Furusawa, “Demonstration of cluster state shaping and quantum erasure for continuous variables,” *Physical Review A* **82**, 032305 (2010).
- [24] S. Yokoyama, R. Ukai, J. Yoshikawa, P. Marek, R. Filip, and A. Furusawa, “Nonlocal quantum gate on quantum continuous variables with minimal resources,” *Physical Review A* **90**, 012311 (2014).
- [25] Radim Filip, “Continuous-variable quantum nondemolishing interaction at a distance,” *Physical Review A* **69**, 052313 (2004).
- [26] R. Filip, P. Marek, and U.L. Andersen, “Measurement-induced continuous-variable quantum interactions,” *Physical Review A* **71**, 042308 (2005).
- [27] K. Hammerer, A.S. Sørensen, and E.S. Polzik, “Quantum interface between light and atomic ensembles,” *Reviews of Modern Physics* **82**, 1041 (2010).
- [28] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, “Entangling macroscopic oscillators exploiting radiation pressure,” *Physical Review Letters* **88**, 120401 (2002).
- [29] M.J. Hartmann and M.B. Plenio, “Steady state entanglement in the mechanical vibrations of two dielectric membranes,” *Physical Review Letters* **101**, 200503 (2008).
- [30] A. Xuereb, C. Genes, and A. Dantan, “Strong coupling and long-range collective interactions in optomechanical arrays,” *Physical Review Letters* **109**, 223601 (2012).
- [31] M. J. Woolley and A. A. Clerk, “Two-mode back-action-evading measurements in cavity optomechanics,” *Physical Review A* **87**, 063846 (2013).
- [32] H. Seok, L. F. Buchmann, E. M. Wright, and P. Meystre, “Multimode strong-coupling quantum optomechanics,” *Physical Review A* **88**, 063850 (2013).
- [33] Huatang Tan, Gaoxiang Li, and P. Meystre, “Dissipation-driven two-mode mechanical squeezed states in optomechanical systems,” *Physical Review A* **87**, 033829 (2013).
- [34] M. J. Woolley and A. A. Clerk, “Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir,” *Physical Review A* **89**, 063805 (2014).
- [35] H. Flayac, M. Minkov, and V. Savona, “Remote macroscopic entanglement on a photonic crystal architecture,” *Physical Review A* **92**, 043812 (2015).
- [36] Ch.-J. Yang, J-H. An, W. Yang, and Y. Li, “Generation of stable entanglement between two cavity mirrors by squeezed-reservoir engineering,” *Physical Review A* **92**, 062311 (2015).
- [37] A. Mari, A. Farace, N. Didier, V. Giovannetti, and R. Fazio, “Measures of quantum synchronization in continuous variable systems,” *Physical Review Letters* **111**, 103605 (2013).
- [38] L. Ying, Y.-Ch. Lai, and C. Grebogi, “Quantum manifestation of a synchronization transition in optomechanical systems,” *Physical Review A* **90**, 053810 (2014).
- [39] T. Weiss, A. Kronwald, and F. Marquardt, “Noise-induced transitions in optomechanical synchronization,” *New Journal of Physics* **18**, 013043 (2016).
- [40] J. Zhang, K. Peng, and S.L. Braunstein, “Quantum-state transfer from light to macroscopic oscillators,” *Physical Review A* **68**, 013808 (2003).
- [41] M. Pinard, A. Dantan, D. Vitali, O. Arcizet, T. Briant, and A. Heidmann, “Entangling movable mirrors in a double-cavity system,” *Europhys. Lett.* **72**, 747 (2005).
- [42] L. Mazzola and M. Paternostro, “Activating optomechanical entanglement,” *Sci. Rep.* **1**, 199 (2011).
- [43] S. Pirandola, D. Vitali, P. Tombesi, and S. Lloyd, “Macroscopic entanglement by entanglement swapping,” *Physical Review Letters* **97**, 150403 (2006).
- [44] K. Børkje, A. Nunnenkamp, and S. M. Girvin, “Proposal for entangling remote micromechanical oscillators via optical measurements,” *Physical Review Letters* **107**, 123601 (2011).
- [45] M. Abdi, S. Pirandola, P. Tombesi, and D. Vitali, “Entanglement swapping with local certification: Application to remote micromechanical resonators,” *Physical Review Letters* **109**, 143601 (2012).
- [46] S.G. Hofer and K. Hammerer, “Entanglement-enhanced time-continuous quantum control in optomechanics,” *Physical Review A* **91**, 033822 (2015).
- [47] M. Schmidt, M. Ludwig, and F. Marquardt, “Optomechanical circuits for nanomechanical continuous variable quantum state processing,” *New Journal of Physics* **14**, 125005 (2012).
- [48] K. R. Brown, C. Ospelkaus, Y. Colombe, A. C. Wilson, D. Leibfried, and D. J. Wineland, “Coupled quantized mechanical oscillators,” *Nature* **471**, 196 (2011).
- [49] Seán M. Meenehan, Justin D. Cohen, Gregory S. MacCabe, Francesco Marsili, Matthew D. Shaw, and Oskar Painter, “Pulsed Excitation Dynamics of an Optomechanical Crystal Resonator near Its Quantum Ground State of Motion,” *Physical Review X* **5**, 041002 (2015), arXiv: 1503.05135.
- [50] C. F. Ockeloen-Korppi, E. Damskågg, J.-M. Pirkkalainen, T. T. Heikkilä, F. Massel, and M. A. Sillanpää, “Low-noise amplification and frequency conversion with a multiport microwave optomechanical device,” arXiv:1602.05779 [cond-mat, physics:quant-ph] (2016), arXiv: 1602.05779.
- [51] Alessio Serafini, Fabrizio Illuminati, and Silvio De Siena, “Symplectic invariants, entropic measures and correlations of Gaussian states,” *Journal of Physics B: Atomic, Molecular and Optical Physics* **37**, L21 (2004).
- [52] C. K. Law, “Interaction between a moving mirror and radiation pressure: A Hamiltonian formulation,” *Physical Review A* **51**, 2537–2541 (1995).
- [53] Vladimir B. Braginsky, Yuri I. Vorontsov, and Kip S. Thorne, “Quantum Nondemolition Measurements,” *Science* **209**, 547–557 (1980).
- [54] A. A. Clerk, F. Marquardt, and K. Jacobs, “Back-action evasion and squeezing of a mechanical resonator using a cavity detector,” *New Journal of Physics* **10**, 095010 (2008).
- [55] D. F. Walls and Gerard J. Milburn, *Quantum Optics* (Springer Science & Business Media, 2007).
- [56] Tobias Eberle, Sebastian Steinlechner, Jöran Bauchrowitz, Vitus Händchen, Henning Vahlbruch, Moritz Mehmet, Helge Müller-Ebhardt, and Roman Schnabel, “Quantum Enhancement of the Zero-Area Sagnac Interferometer Topology for Gravitational Wave Detection,” *Physical Review Letters* **104**, 251102 (2010).

- [57] Jasper Chan, T. P. Mayer Alegre, Amir H. Safavi-Naeini, Jeff T. Hill, Alex Krause, Simon Groeblacher, Markus Aspelmeyer, and Oskar Painter, “Laser cooling of a nanomechanical oscillator into its quantum ground state,” *Nature* **478**, 89–92 (2011), arXiv:1106.3614 [quant-ph].
- [58] J. D. Teufel, T. Donner, Dale Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, “Sideband cooling of micromechanical motion to the quantum ground state,” *Nature* **475**, 359–363 (2011).
- [59] Ying Dong, Keye Zhang, Francesco Bariani, and Pierre Meystre, “Work measurement in an optomechanical quantum heat engine,” *Physical Review A* **92**, 033854 (2015), arXiv: 1504.02901.
- [60] Keye Zhang, Francesco Bariani, and Pierre Meystre, “Theory of an optomechanical quantum heat engine,” *Physical Review A* **90**, 023819 (2014), arXiv: 1406.5160.
- [61] Cyril Elouard, Maxime Richard, and Alexia Auffèves, “Reversible work extraction in a hybrid opto-mechanical system,” *New Journal of Physics* **17**, 055018 (2015), arXiv: 1309.5276.
- [62] M. Brunelli, A. Xuereb, A. Ferraro, G. De Chiara, N. Kiesel, and M. Paternostro, “Out-of-equilibrium thermodynamics of quantum optomechanical systems,” *New Journal of Physics* **17**, 035016 (2015), arXiv: 1412.4803.