Transfer of non-Gaussian quantum states of mechanical oscillator to light

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Non-Gaussian quantum states are key resources for quantum optics with continuous-variable oscillators. The non-Gaussian states can be deterministically prepared by a continuous evolution of the mechanical oscillator isolated in a nonlinear potential. We propose feasible and deterministic transfer of non-Gaussian quantum states of mechanical oscillators to a traveling light beam, using purely all-optical methods. The method relies on only basic feasible and high-quality elements of quantum optics: squeezed states of light, linear optics, homodyne detection, and electro-optical feedforward control of light. By this method, a wide range of novel non-Gaussian states of light can be produced in the future from the mechanical states of levitating particles in optical tweezers, including states necessary for the implementation of an important cubic phase gate.

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I. INTRODUCTION

During the last three decades, quantum optics tested a number of nonclassical physical phenomena with continuous variables of light. This was possible thanks to the development of quantum features of nonlinear optics [1]. It mainly enabled a deep examination of quadratic nonlinear effects at the quantum level. The quadratic nonlinearities are generically capable of producing squeezed states of light [2]. They have direct experimental applications in quantum metrology [3], quantum cryptography [4], and quantum computation [5]. Using the squeezed states from optical parametric oscillators (OPOs) as available offline resources [6], it is possible to induce any quadratic nonlinearities on any quantum state of light [7,8]. They have been extensively experimentally investigated [9–13]. They exploit the achievements of quantum optics during the last decade, mainly the high quality and stability of linear optical interferometry, the high efficiency and low noise of homodyne detection, and the high speed and precision of the electro-optical feedforward control of light [6].

However, the quadratic nonlinearities provided by OPOs are already not sufficient for future applications. A highly required non-Gaussian quantum state is a state from a cubic nonlinearity. It is a key resource for deterministic implementation of a basic cubic phase gate [14–17]. This cubic gate is needed to finally complete the set of existing linear and quadratic quantum operations. This complete set is then in principle sufficient to create any quantum nonlinearity. Recently, the approximate state from a weak cubic nonlinearity has been conditionally prepared by all-optical approach [18,19]. However, this method cannot be deterministic and does not allow one to prepare states from stronger cubic nonlinearities.

Currently developing quantum optomechanics [20–22] is a very good candidate to surpass this limitation. A quantum-mechanical oscillator represented by a levitating particle can evolve under the influence of an external highly nonlinear potential, for example the cubic potential, in optical tweezers [23–27]. The mechanical motion can shortly feel a strong nonlinear potential and, therefore, it can deterministically generate a highly non-Gaussian quantum state of the mechanical oscillator. The optomechanical experiments with levitating particles are currently progressing in this direction; however, the experimental setup for such a generator still remains open as a future target. At the classical level, few steps towards such possibilities have been experimentally tested inside the optical tweezers [28–33].

A quantum-mechanical oscillator initially in a ground state subjected to a nonlinear potential higher than quadratic evolves to a quantum non-Gaussian state. These states exhibit an important structure of negative areas of Wigner function in phase space [34–37]. However, that negativity is very sensitive to both the mechanical decoherence and the efficiency of the state transfer from mechanical oscillator to light. To avoid mechanical decoherence or other mechanical noise, the preparation of the mechanical state has to be faster than any decoherence process and also a pulse of light reading the mechanical state has to interact only briefly with the mechanical oscillator. The short interaction time can cause limited efficiency of transfer, which can strongly suppress or completely vanish the negativity of the Wigner function. Recently, the first successful conversion of a basic nonclassical Gaussian quantum state from mechanical motion to microwave radiation was experimentally tested [38,39]. It has opened a chance to generally investigate a realistic regime of beam-splitter-type conversion based on that experiment. However, the requirements to transfer the non-Gaussian states with negative Wigner functions in that realistic regime are much more strict. It is therefore important to develop very efficient conversion of the non-Gaussian quantum state of the mechanical oscillator to light before the mechanical nonlinearity is experimentally investigated in the future.

In this paper, we propose a high-quality conversion of non-Gaussian quantum states from a mechanical oscillator to a traveling light beam powered by a feasible squeezing of light produced by the available OPOs. If the conversion efficiency of the beam-splitter type of optomechanical coupling is limited, the proposed converter transfers any state of the mechanical oscillator to light without any further access to the mechanical oscillator. The proposed idea uses the common Gaussian entangled states of light produced by the OPOs and injected into the input of optomechanical systems, together with an all-optical scheme at the output. It does not require any further development of the mechanical oscillator. The optical converter uses only routine high-quality quantum-optical elements: auxiliary squeezed states of light, linear optics, highly efficient homodyne detection, and electro-optical feedforward control.

The conversion approaches a perfect transfer as the auxiliary squeezing of light increases. Further, we prove a key stability of the conversion under weak incoupling and outcoupling losses presented before and after the optomechanical interaction. The effective model of incoupling and outcoupling losses describing high-quality converters is motivated by the only existing conversion process for nonclassical states [38,39]. The motivating example of an important quantum state from a cubic nonlinearity is analyzed. The proposed Gaussian converter opens up many future possibilities for the generation of a wide range of non-Gaussian quantum states of light, after they are reached in the mechanical oscillator. It can be used also for other purposes, for example to efficiently read out quantum memories.

The paper is organized as follows. In Sec. II, the direct converter based on a beam-splitter type of optomechanical coupling is described. This section is accompanied by the Appendix, where a negligible impact of the mechanical bath during the optomechanical coupling is verified and an effective model of the incoupling and outcoupling efficiencies is technically introduced. In Sec. III, the high-quality converter powered by squeezed states of light is proposed and stability of the conversion under limited incoupling and outcoupling efficiencies is verified. In Sec. IV, the transfer of negativity of the Wigner function is analyzed, followed by analysis of the transfer of quantum states generated in the cubic nonlinear potential in Sec. V. To enhance the negativity of the Wigner function, the effects of presqueezing of the mechanical oscillator are described in Sec. VI. The conclusion briefly summarizes the results. In the Appendix, we present a technical derivation of the direct beam splitter with the incoupling and outcoupling efficiencies which can be used to effectively describe the high-quality conversion processes tested experimentally in Refs. [38,39].

II. DIRECT MECHANICAL-TO-OPTICAL CONVERTER

To describe the conversion mechanism for a broad community of quantum optics and quantum optomechanics, we consider a standard optomechanical interaction in the pulsed regime [40,41]. We skip the technical details of various implementations. Moreover, we use here the effective and simple model sufficient to describe high-quality conversion from mechanics to radiation used in the experiment [38,39]. This model can be obtained from full analysis of pulsed optomechanical systems with both optical and mechanical baths presented in the Appendix.

To simply describe a core of the ideal conversion process, we consider a mechanical oscillator with frequency ω_m and without any decay. On the other hand, we assume a radiation mode in a cavity with resonance frequency ω_c and cavity decay rate κ. A signal optical pulse of duration $\Delta\tau$ and carrier frequency ω_l enters the cavity and interacts by radiation pressure with the mechanical oscillator. The goal is to read out any quantum state of the mechanical oscillator. In the regime of strong optical pumping, optomechanical interaction can be described by a linearized coupling with a strength g and laser detuning $\Delta_c = \omega_c - \omega_l$. Moreover, for frequency-resolved sidebands and weak-coupling regime $g \ll \kappa \ll \omega_m$, a red-detuned signal pulse ($\Delta_c = \omega_m$) inside the time interval $(\tau,\tau+\Delta\tau)$ can advantageously feel a beam-splitter interaction described in the rotating-wave approximation by Langevin equations

$$\dot{a}_c = -\kappa a_c - i g a_m - \sqrt{2\kappa}\, a_{\text{in}}, \quad \dot{a}_m = -i g a_c, \tag{1}$$

where a_c, a_{in} (a_m) are annihilation operators rotating (counter-rotating) with frequency ω_m [40]. The output optical field is then described by the input-output relation $a_{\text{out}} = a_{\text{in}} + \sqrt{2\kappa}\, a_c$. For operators

$$A_{\text{in}} = -i\sqrt{\frac{2G}{e^{2G\Delta\tau}-1}} \int_{\tau}^{\tau+\Delta\tau} e^{Gt} a_{\text{in}}(t) dt,$$
$$A_{\text{out}} = i\sqrt{\frac{2G}{1-e^{-2G\Delta\tau}}} \int_{\tau}^{\tau+\Delta\tau} e^{-Gt} a_{\text{out}}(t) dt \tag{2}$$

satisfying $[A_j, A_j^\dagger] = 1$, $j = \text{in},\text{out}$, and $B_{\text{in}} = a_m(\tau)$ and $B_{\text{out}} = a_m(\tau+\Delta\tau)$, the beam-splitter interaction

$$A_{\text{out}} = \sqrt{T} B_{\text{in}} + \sqrt{1-T} A_{\text{in}},$$
$$B_{\text{out}} = \sqrt{T} A_{\text{in}} - \sqrt{1-T} B_{\text{in}}, \tag{3}$$

between the exponentially rising and decreasing temporal modes of light, and the modes of the mechanical oscillator can be adjusted, where $G = g^2/\kappa$ and $T = 1 - e^{-2g^2\Delta\tau/\kappa}$ is a conversion efficiency of transfer from the mechanical mode to the optical one. We are concerned mainly about the first of Eqs. (3). For a time duration $\Delta\tau$ substantially shorter to avoid mechanical decoherence and all other technical noises, the beam-splitter coupling allows a partial, but almost unitary, coupling between the mechanical mode and the output mode of the cavity. We verified the almost-unitary beam-splitter coupling by taking the mechanical decoherence and noise into account beyond the adiabatic elimination of the cavity mode; see the Appendix. When mechanical decoherence is fast or other destructive effects appear already for small $g^2\Delta\tau/\kappa$, the conversion efficiency T becomes limited. For any $T < 1$, the conversion reduces the negativity of the Wigner function of the mechanical state during its transfer to light. For $T < 0.5$, the negativity of the Wigner function of the mechanical state cannot be transmitted to light at all. It is because any Wigner function of the state after $T = 0.5$ becomes equal to the positive Husimi Q function of that state [42].

The simple coupling (3) of light and the optomechanical cavity can suffer from imperfections caused by additional damping in both optical and mechanical systems. We model these imperfections by effective in- and outcoupling losses which include as well other losses in the external parts of the converter. The numerical analysis of the full dynamics of the system carried out in the Appendix proves that the model of losses approximates the imperfections very well for the setup reported in Refs. [38,39]. This system is the only one to the best of our knowledge that was experimentally capable of transferring nonclassical states from mechanics to light.

Although the losses can be seemingly small, they may seriously limit the quality and stability of any improvement of the converter. The losses change the simple conversion (3) to the following transformation:

$$A_{\text{out}} = \sqrt{T\eta_o} B_{\text{in}} + \sqrt{T_L} A_{\text{in}} + \sqrt{1 - T\eta_o - T_L} A_0, \tag{4}$$

where $T\eta_o$ and $T_L = (1-T)\eta_i\eta_o$ are transmission efficiencies of states of mechanical and radiation modes through the direct converter, η_i, η_o are the incoupling and outcoupling transmissions, and A_0 stands for the auxiliary annihilation operator of a vacuum mode. Numerical simulations prove the validity of Eq. (4) for the recent electromechanical experiment [38,39] (see the Appendix). We, however, consider T, η_i, and η_o as general parameters in the following discussion to keep our results as general as possible. If the input light mode described by $A_{\rm in}$ is in the vacuum state, the transfer only suffers from additional outcoupling efficiency η_o. The goal is to reach a beam-splitter type of converter with, at least, conversion efficiency $T' > T\eta_o$ and, later, with T' as close to unity as possible. A basic benchmark for such a task is clearly $T' > 0.5$, when the negative Wigner function of pure mechanical states can be transferred to light. Moreover, the stability of any conversion efficiency T' under a small decrease η_i and η_o from unity is a key issue which has to be analyzed.

III. CONVERTER POWERED BY SQUEEZED LIGHT

To build a better beam-splitter type of converter with $T' > \eta_o T$, a linear optical scheme with homodyne detectors and electro-optical feedforward control depicted in Fig. 1 is proposed. It uses the squeezed states of light, a broadly available resource in quantum optics. Currently, the squeezing of light can be safely larger than -9 dB [43], corresponding to the reduction of the quadrature variance to $V_S = 0.125$ from the vacuum variance calibrated to $V_0 = 1$. The Gaussian two-mode entangled state can be obtained from two orthogonally squeezed lights, generated by the OPOs, which are mixed at a symmetrical 50:50 beam splitter (SBS) implementing the transformation

$$A_{\rm in} = \frac{1}{\sqrt{2}}(A_{S1} + A_{S2}), \quad A_{\rm anc} = \frac{1}{\sqrt{2}}(A_{S2} - A_{S1}), \tag{5}$$

where A_{Si} are annihilation operators of orthogonally squeezed modes and $A_{\rm anc}$ is the annihilation operator of the ancillary optical mode. The mode described by $A_{\rm in}$ is then injected into the cavity. After the optomechanical coupling (4), the mode

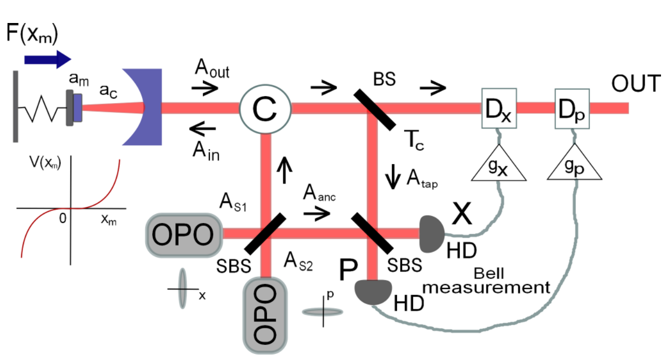

FIG. 1. (Color online) Universal conversion of non-Gaussian quantum states of the mechanical oscillator generated by a nonlinear potential to traveling light: OPO, optical parametric oscillator generating squeezed light; C, optical circulator; SBS, balanced 50:50 beam splitter; BS, beam splitter with transmittance T_c; HD, homodyne detector; g_x, g_p, electronic units with variable gains; and D_x, D_p, displacement operations.

described by output operator $A_{\rm out}$ is then split at a beam splitter (BS) to two modes described by the operators

$$\begin{aligned} A'_{\rm out} &= \sqrt{T_c}A_{\rm out} - \sqrt{1-T_c}A_{\rm vac}, \\ A_{\rm tap} &= \sqrt{T_c}A_{\rm vac} + \sqrt{1-T_c}A_{\rm out}, \end{aligned} \tag{6}$$

where $A'_{\rm out}$ is the output annihilation operator and $A_{\rm tap}$ is the annihilation operator of the tapped mode. The tapped mode is then jointly measured with the ancillary mode in the standard high-quality continuous-variable Bell measurement [6,44,45]. The Bell measurement mixes $A_{\rm tap}$ and $A_{\rm anc}$ at the SBS and measures the outputs in complementary variables X and P to obtain a complex number $\bar{\alpha} = \frac{1}{\sqrt{2}}(A_{\rm tap} - A^\dagger_{\rm anc})$. The complex number is then used to control displacements of the light mode $A'_{\rm out}$ in the following way: $A''_{\rm out} = A'_{\rm out} + g_c\bar{\alpha}$. The displacement is performed by standard high-speed and very precise electro-optical feedforward control [6,46], where electronic gains g_x and g_p can be freely optimized. The methodology of the standard calculations can be found in Refs. [6,46]. For given η_i, η_o, and T, we can find optimal transmittance and electronic gain,

$$T_c = 1 - \eta_i\eta_o(1-T), \quad g_{x,p} = \sqrt{2\frac{\eta_i\eta_o(1-T)}{1-\eta_i\eta_o(1-T)}}, \tag{7}$$

of the control circuit, to reach the beam-splitter coupling

$$A''_{\rm out} = \sqrt{T'}B_{\rm in} + \sqrt{1-T'}A_0 + \sqrt{\frac{1-T_c}{T_c}}(A_{\rm in} - A^\dagger_{\rm anc}) \tag{8}$$

with the transmittance

$$T' = \frac{\eta_o T}{1-\eta_i\eta_o(1-T)}. \tag{9}$$

The second term in Eq. (8), with A_0 corresponding to a mode in the vacuum state, represents the minimum quantum noise for the transmittance T'. The operator part of the third term can be rewritten as $A_{\rm in} - A^\dagger_{\rm anc} = \frac{1}{\sqrt{2}}(X_{S1} + iP_{S2})$, where $X_{S1} = A_{S1} + A^\dagger_{S1}$ and $P_{S2} = (A_{S2} - A^\dagger_{S2})/i$ are quadrature operators of two auxiliary squeezed modes of light. As the variances $V_S = \langle X^2_{S1}\rangle = \langle P^2_{S2}\rangle$ decrease, residual additive noise caused by the third term in Eq. (8) vanishes.

The beam-splitter coupling (8) can be then written for the generalized output operators $Q = X, P$ of quadratures of light as

$$Q = \sqrt{T'}Q_M + \sqrt{1-T'}Q_0 + Q_N, \quad V'_N = \frac{(1-T_c)}{T_c}V_S, \tag{10}$$

where $Q_M = X_M, P_M$ are position and momentum operators of the mechanical oscillator, $Q_0 = X_0, P_0$ are noisy operators of a virtual oscillator at the ground state, and $Q_N = X_N, P_N$ are noisy operators of a virtual oscillator with the same variance V'_N. The variance V'_N vanishes, as the Gaussian states with larger squeezing are produced from both OPOs in Fig. 1. Using Eqs. (7), for larger $\eta_i\eta_o(1-T)$ appearing when T decreases, smaller V_S is required. Remarkably, using only Gaussian squeezed states we can better convert non-Gaussian states with the conversion transmittance T' larger than the original transmittance $T\eta_o$ for any $\eta_i, \eta_o > 0$. For the high

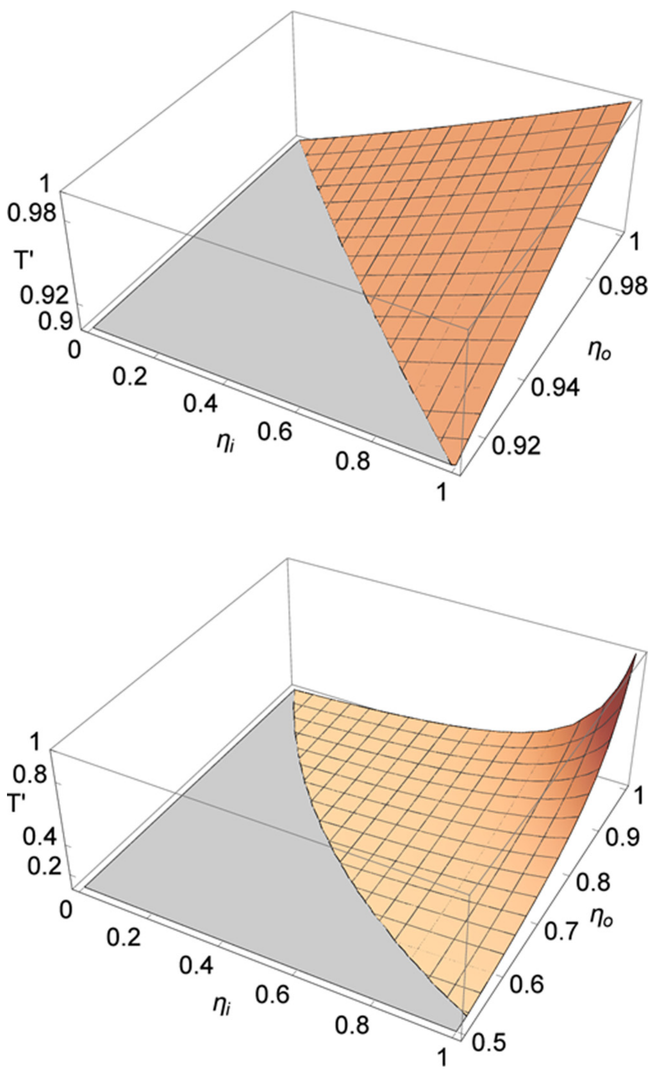

FIG. 2. (Color online) The improved transmittancy $T' > T$ of the squeezed-light powered converter for good converter $T = 0.9$ (top) and bad converter $T = 0.1$ (bottom). η_i,η_o are incoupling and outcoupling efficiencies.

incoupling and outcoupling efficiencies $\eta_i \approx 1$ and $\eta_o \approx 1$, the achievable transmittance approaches

$$T' \approx 1 - \frac{1-\eta_o}{T} - \frac{(1-T)(1-\eta_i)}{T}. \tag{11}$$

For a high-quality direct converter with $T \approx 1$, η_o is mainly limiting whereas the conversion is much more tolerant to a smaller η_i, as is visible in Fig. 2 (top). If the incoupling efficiency η_i can be neglected, for any $\eta_o > 1/(1+T)$ the transmittancy $T' > \eta_o T$ surpasses the direct mechanical converter. For the high-quality converters, the squeezed light can be therefore advantageously used to approach the maximal conversion efficiency $T' = \eta_o$. On the other hand, for a low-quality converter with $T \ll 1$, the converter is sensitive to both η_i and η_o more equally. Transmittancy T' can be improved relatively very much; however, it still might not be sufficient to reach T' large enough to transfer highly nonclassical states, as is visible from Fig. 2 (bottom). In Fig. 3, we illustratively demonstrate the relative improvement in the transmittancy T'

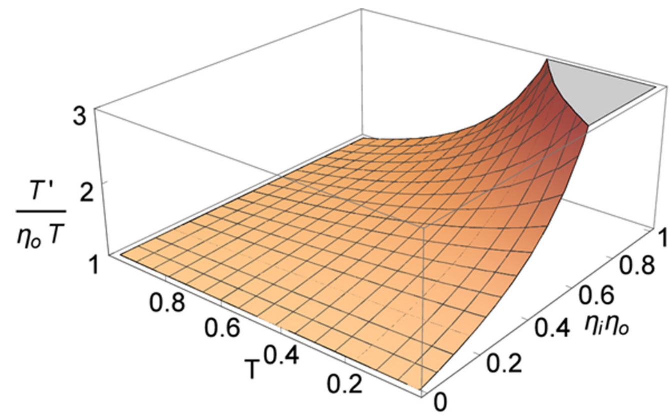

FIG. 3. (Color online) The relative improvement between the transmittancy T' and the transmittancy $\eta_o T$ of the direct converter. η_i, incoupling efficiency, and η_o, outcoupling efficiency.

over the direct transmittancy $\eta_o T$. The threshold to observe the negative Wigner function is discussed in the next section.

Remarkably, no challenging non-Gaussian error correction [47] is required to improve the transfer of non-Gaussian states from mechanical systems to light. Moreover, the conversion scheme does not produce any excess noise when the incoupling and outcoupling efficiencies are present. It therefore overcomes the interfaces based on teleportation with squeezed states [6], where the incoupling and outcoupling loss introduces the excess noise destroying fragile quantum effects from the higher-order nonlinearities.

IV. CONVERSION OF NEGATIVITY OF THE WIGNER FUNCTION

From the Hudson theorem [48] it follows that any pure non-Gaussian state exhibits negativity of the Wigner function. To reach $T' > 0.5$ converting the negativity of any pure state in the limit of small V_S, the condition

$$T > \frac{1-\eta_i\eta_o}{(2-\eta_i)\eta_o} \tag{12}$$

has to be fulfilled. In Fig. 4, we demonstrate the required T for transmission of multiple negativities of the Wigner function for the decreasing incoupling and outcoupling efficiencies η_i and η_o. For $\eta_i = 1$ and $\eta_o < 1$ (vanishing intracavity losses), Eq. (12) simplifies to $T > (1-\eta_o)/\eta_o$; therefore, $\eta_o > 0.5$ is generally required. To reach $T' > 0.5$ for $T < 0.5$, it needs $\eta_o > 2/3$. For symmetrical $\eta = \eta_i = \eta_o$, condition (12) simplifies to $T > T_{\text{th}} = \frac{1-\eta^2}{(2-\eta)\eta}$, where $T_{\text{th}} < 2(1-\eta)$. It follows, if $T > 2(1-\eta)$, that the presence of negativity of the Wigner function in the traveling output beam can be guaranteed for any pure mechanical non-Gaussian quantum state. It can be compared with the condition $T > 1/(2\eta)$ for the preservation of negativity without the proposed method. Clearly, for $\eta \approx 1$, $T' > T$ is obtained almost for all $T < 0.5$. It means the transmittance $T < 0.5$ can be efficiently increased to preserve the negative Wigner function, although the in- and outcoupling efficiency η is not exactly unity. If η_i is different from η_o, then $\eta_o > 0.5$ is required to preserve the negativity of the Wigner function. On the other hand, $\eta_i < 0.5$ is tolerable

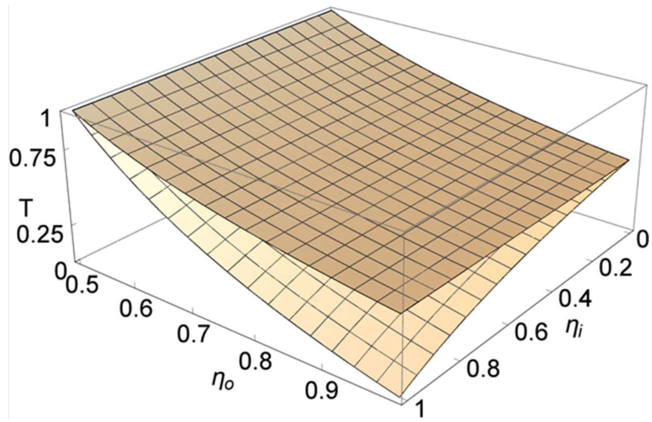

FIG. 4. (Color online) The minimal transmittance T of beam-splitter coupling to reach $T' > 0.5$ for the squeezed-state powered converter visualized by the lower light plane. η_i, incoupling efficiency, and η_o, outcoupling efficiency. The dark upper plane corresponds to the condition $T > 1/(2\eta_o)$ for the direct converter without the proposed method.

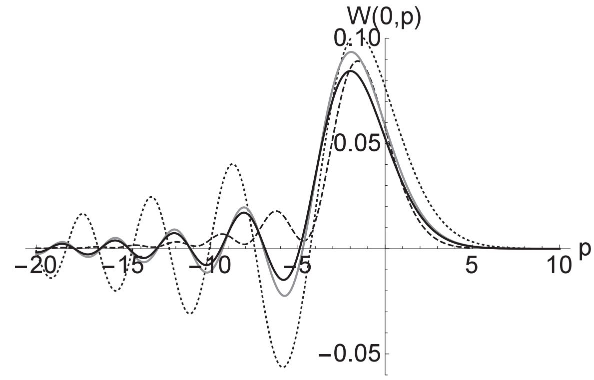

FIG. 5. Wigner function $W(0,p)$ of state from cubic mechanical nonlinearity with $\kappa_3\tau = 1$ transferred by universal converter (solid black line) $T = 0.5$, $\eta_i = \eta_o = 0.9$, variance $V_S = 0.125$ and for ground initial state : (solid gray line) $T = 0.5$, $\eta_i = \eta_o = 0.9$ and asymptotic variance $V_S = 0$. For comparison, (dashed line) without universal converter $T = 0.5$, $\eta_o = 0.9$, (dotted line) ideal state from cubic nonlinearity with $T = 1$ and $\eta_o = 1$.

to keep the negativity of the Wigner function. Clearly, the proposed method allows one to transfer the negativity of the Wigner function generated in nonlinear dynamics of the mechanical oscillator even for $T < 0.5$, if η_i and η_o are sufficiently large.

V. CONVERSION OF MECHANICAL CUBIC STATE

To transfer the non-Gaussian quantum state from the cubic nonlinearity to a traveling optical beam, we consider first that the mechanical oscillator was cooled down to the mechanical ground state $|0\rangle_M$ at the time $t = 0$. After the ground-state preparation, the cubic potential $V(x_m) = \frac{1}{3}\kappa_3 x_m^3$ starts to influence the dynamics of the mechanical oscillator. To evaluate only the impact of the conversion on the nonclassical state arising in the cubic potential, we neglect the mechanical decoherence. It is naturally required to obtain highly nonclassical mechanical states. A feasibility analysis of the realistic generation of the cubic states is beyond the scope of this paper.

For a time interval τ shorter than the time of mechanical decoherence, the dynamics can be described by the Heisenberg equations of motion,

$$\dot{x}_m = \omega_m p_m, \quad \dot{p}_m = -\omega_m x_m - 6\kappa_3 x_m^2, \tag{13}$$

for position and momentum operators $x_m = a_m + a_m^\dagger$ and $p_m = (a_m - a_m^\dagger)/i$. For $\omega_m \ll \kappa_3|x_m|$ and short time duration τ, the dynamics (13) is predominantly determined by the cubic nonlinearity. In a first order of short-time approximation, the mechanical position x_m remains constant: $x_m(\tau) \approx x_m(0)$ [41]. We neglect simultaneously the mechanical damping and harmonic motion, which complicate the dynamics. The quantum-mechanical momentum evolves as $p_m(\tau) \approx p_m(0) - 6\kappa_3\tau x_m^2(0)$. Since large κ_3 can be used in the nonlinear potential, a relatively large cubic effect can be obtained. Further, it can be increased by presqueezing of the mechanical state. We can therefore approximate the state after evolution as $\exp(i\kappa_3\tau x_m^3)|0\rangle_M$. To witness the negativity of the Wigner function, we determine a relevant slice of the Wigner function,

$$W_0(0,p_m) = \frac{1}{2\sqrt{2}\pi^{\frac{3}{2}}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}-2i\kappa_3\tau y^3 - ip_m y}dy, \tag{14}$$

for $x_m = 0$ at time τ. The numerical evaluation of this integral for $p_m > 0$ gives positive values of $W_0(0,p_m)$, contrary to the oscillating behavior of $W_0(0,p_m)$ for $p_m < 0$ where it not-periodically reaches an uncountable number of negative semicircles, as depicted in Fig. 5 (dotted line). The amplitudes of oscillations are decreasing in the amplitude and increasing in the frequency. The oscillations are present for any $\kappa_3\tau > 0$ and the amplitudes become larger when $\kappa_3\tau$ increases. The multiple negative values of the oscillating Wigner function manifest the highly quantum non-Gaussian character of the state from cubic nonlinearity, incompatible with any mixture of Gaussian states from quadratic nonlinearities [49].

After the transfer of the mechanical state to the output optical mode through the beam-splitter coupling (10), the cut of the Wigner function

$$W_0(0,p) = \int_{-\infty}^{\infty} \frac{e^{-\frac{(1+V_N')y^2}{2T'}-2i\kappa_3 t y^3 - i\frac{py}{\sqrt{T'}}}dy}{\sqrt{8\pi^3 T'(1+V_N'+12i\kappa_3(1-T'+V_N')y)}}$$

of the optical mode leaving the converter at Fig. 1 preserves oscillations, but $W_0(0,p)$ becomes purely positive when $V_N' > 2T' - 1$. The latter condition is equivalent to that for the transfer of the Fock state $|1\rangle_M$ from mechanical oscillator to light. Considering Eqs. (7) and (9), we finally obtain the condition

$$V_S < 1 - \frac{1 - 2\eta_o T}{\eta_i\eta_o(1-T)} \tag{15}$$

for transfer of the multiple negativities of the Wigner function of the cubic state generated in the mechanical oscillator to the traveling light beam. Clearly, for $\eta_o T > 0.5$ the method is not required if our purpose is just to observe the negative Wigner function at the optical output. To be able to transmit the negativity of the Wigner function with the help of any squeezing, the transmittancy has to satisfy

$$T > \frac{1-\eta_i\eta_o}{(2-\eta_i)\eta_o}. \quad (16)$$

Necessarily, $\eta_o > 0.5$ is required. For a high-quality direct interface with η_i and η_o close to unity, the maximal variance can be approximated by

$$V_{S,\max} \approx \frac{T}{1-T} - \left(2 - \frac{1}{1-T}\right)(1-\eta_i) - \frac{1-\eta_o}{1-T}. \quad (17)$$

For the transmittancy $T < 0.5$, when the direct interface cannot transmit the negativity of the Wigner function, it is necessary to reach at least $V_S < T/(1-T)$. For the feasible variance $V_S = 0.125$ (-9 dB), we can only correct the transmittancy $T > 0.11$, which is, however, enough to be reached by the high-quality direct interfaces. For $\eta_i = \eta_o = 0.9$, the transmittance $T > 0.27$ is sufficient to transmit for the same squeezing $V_S = 0.125$. Clearly, the feasible squeezed light generated in the experiments is sufficient to improve the transfer for high-quality interfaces with $T < 0.55$. To reach transfer of negativity for very small T, the variance V_S of the squeezed state has to decrease below

$$V_{S,\max} \approx 1 - \frac{1}{\eta_i\eta_o} + \frac{2\eta_o - 1}{\eta_i\eta_o}T, \quad (18)$$

which is possible only for η_i and η_o sufficiently close to unity. Squeezed states of light are still very helpful, but only if the converter has reasonably small incoupling and outcoupling losses. On the other hand, for high $\eta_0 T > 0.5$ the negativity of the Wigner function is transferred always; however, the method can help to approach the oscillations of the Wigner function more precisely. Naturally, the transmittancy cannot be improved over η_o, which becomes the main limit of quality of the converter.

The numerical example is depicted in Fig. 5. It clearly illustrates that all multiple oscillations of negative Wigner function can be restored if they vanish (dashed line), despite larger values of the incoupling and decoupling losses. In Fig. 6, the contours of regions where the Wigner function is negative are plotted for different T and $\eta = \eta_i = \eta_o$. As η increases, the regions of negativity can be well transferred from the mechanical oscillator to light. They are preserved and only shifted due to a residual damping in the converter powered by the squeezed light. It is an indicator that highly nonclassical states produced in mechanical systems can be efficiently transferred to light with the help of squeezed states of light, homodyne detection, and electro-optical feedforward techniques. Remarkably, the examples use an available amount of squeezing from OPOs (solid black line), in comparison with an asymptotic case (solid gray line). The remaining difference from the ideal case becomes smaller, when the incoupling and outcoupling losses are reduced. The effect of cubic nonlinearity can be amplified by presqueezing of the ground state of the mechanical oscillator before it evolves in the cubic potential, as is described in the following section.

VI. CUBIC STATE GENERATION WITH PRESQUEEZING

In the near future, a quantum-mechanical oscillator will be advantageously prepared in a highly squeezed position state.

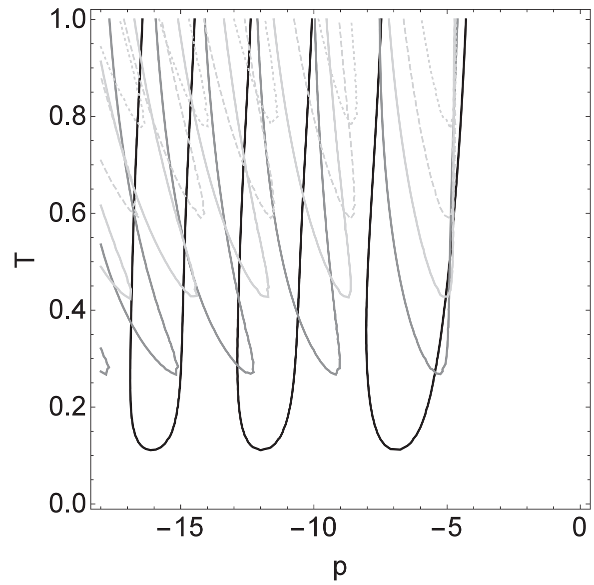

FIG. 6. Contours of regions of negativity of the Wigner function $W(0,p)$ of state from cubic mechanical nonlinearity with $\kappa_3\tau = 1$ transferred by universal converter with T, $\eta = \eta_i = \eta_o$, and variance $V_S = 0.125$: $\eta = 1$ (solid black line), $\eta = 0.9$ (dark gray line), $\eta = 0.8$ (light gray line), $\eta = 0.7$ (dashed light gray line), and $\eta = 0.6$ (dotted light gray line).

An approach to reach this can be a short time application of very deep quadratic potential after the cooling of the mechanical oscillator. The width of such a potential dip can be much narrower than the position variance of the mechanical ground state. Alternatively, a highly squeezed state can be generated by a precise pulsed measurement, which projects the mechanical oscillator almost to the position basis state $|x = \bar{x}_m\rangle_M$. Since $\bar{x}_m$ is known from the measurement result, it can be compensated to approach $|x = 0\rangle_M$. After both types of procedures, the position state $|x = 0\rangle_M$ can be swapped to the state $|p = 0\rangle_M$ by a free evolution of the mechanical oscillator during the time interval $\pi/(2\omega_M)$. After these steps, the cubic nonlinearity can be applied for a period much shorter than the mechanical decoherence time. It allows one to approach the state proportional to $\int_{-\infty}^{\infty} dx\, \exp(i\kappa_3 t x^3)|x\rangle_M$ with the Wigner function

$$W(x,p)\mathrm{Ai}\left[\frac{p + 6\kappa_3 t x^2}{(6\kappa_3 t)^{1/3}}\right],$$

where Ai[z] is the Airy function. The Airy function captures interesting quantum features of the state from quantum cubic nonlinearity. When z is positive, Ai(z) is positive, convex, and decreasing exponentially to zero. When z is negative, Ai(z) oscillates around zero with ever-increasing frequency and ever-decreasing amplitude. The oscillations in $W(0,p)$ appear for any $p < 0$ and they are present for any $\kappa_3 t$, only their amplitudes are smaller. These oscillations are highly nonclassical aspects of continuous time dynamics.

Any physical measurement or physical potential will ideally prepare the physical squeezed state $|r\rangle_M = S_M(r)|0\rangle_M$ in position. The squeezing is applied sufficiently faster than mechanical free evolution and then the state is transformed by the latter to a squeezed state in the momentum. The squeezing amplifies the initial position of the mechanical oscillator $x_m(0) \to x_m(0)\exp(r)$ in the ground state and squeezes the momentum variable $p_m(0) \to p_m(0)\exp(-r)$, before the cubic nonlinear potential is applied. r is an effective squeezing parameter determined as a product of evolution and measurement time and either width of quadratic potential or strength of nondemolition measurement. After fast evolution in the cubic nonlinear potential, we obtain the state

$$S_M^\dagger(r)e^{i\kappa_3 t X_m^3}S_M(r)|0\rangle_M = S_M(r)\kappa_3 e^{i\exp(r)tX_m^3}|0\rangle_M,$$

which is a squeezed version of much faster evolution in the cubic potential. The interaction time t changes to $\exp(r)t$ being enhanced by squeezing factor r. We can therefore decrease the product $\kappa_3 t$ to protect evolution against the mechanical decoherence and noise and imperfections of the potential for the mechanical oscillator far from the origin.

VII. CONCLUSION

We proposed efficient and feasible transfer of the non-Gaussian quantum states of the mechanical oscillator to light. The mechanical part relies on a running development of optically levitating particles in optical tweezers, where a wide range of nonlinear optical potentials can be well designed. The required non-Gaussian states of light can be therefore produced in the near future. Our proposal of the squeezed-state powered converter does not require any modification of the mechanical oscillator; it is purely based on feasible quantum operations on the optical part of the converter. This proposal is an example of how quantum optomechanics can be useful to generate new nonclassical states for quantum optics, using, however, simultaneously high-quality outcomes of quantum optics: squeezed light, homodyne detection, and electro-optical feedforward control. The idea of the proposed converter is, however, more general; it can be applied to other physical platforms [50], for example, experiments with quantum memories [51] or microwave radiation [38,39].

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APPENDIX: INFLUENCE OF THE MECHANICAL BATH ON READOUT OF THE MECHANICAL STATE

The transformation (4) in the main text is obtained by adiabatic elimination of the cavity mode that amounts to putting $\dot{a}_c = 0$ in Eq. (1). This approximation is valid if $G/\kappa \ll 1$ and the interaction time is sufficient ($\Delta\tau \gg 1/\kappa$). These conditions can be combined as $1/\Delta\tau \ll G \ll \kappa$. Simultaneously, we neglected the influence of the optical and mechanical baths. In this Appendix, we provide the analysis of a general optomechanical setup, make numerical estimations using parameters from Refs. [38,39], and compare to the simple model based on the effective incoupling and outcoupling efficiencies that we used in the main text.

We start with writing linearized equations of motion describing an optomechanical system [22]. The two modes comprising the system are coupled to each other by means of a beam-splitter-like interaction and as well interact with the environment. The mechanical mode is coupled to the thermal bath at rate γ. The optical mode is not only coupled to the detection channel at rate κ_e, but also experiences losses at rate κ_i: $\kappa_e + \kappa_i = \kappa$. The equations then read

$$\frac{dv}{dt} = Av - \sqrt{2\kappa_e} f_{\rm in} - f, \tag{A1}$$

where $v = (X,Y,q,p)^T$ is the vector of quadratures, A is the drift matrix, f is the vector of noises, and $f_{\rm in} = (X_{\rm in}, Y_{\rm in}, 0, 0)$ is the vector of optical input fields:

$$A = \begin{pmatrix} -\kappa & 0 & -g & 0 \\ 0 & -\kappa & 0 & -g \\ g & 0 & -\frac{\gamma}{2} & 0 \\ 0 & g & 0 & -\frac{\gamma}{2} \end{pmatrix}, \quad f = \begin{pmatrix} \sqrt{2\kappa_i}X_{\rm in}^L \\ \sqrt{2\kappa_i}Y_{\rm in}^L \\ \sqrt{\gamma}\xi_q \\ \sqrt{\gamma}\xi_p \end{pmatrix}.$$

The equations allow an analytical solution written with the help of the matrix exponential $M(t) \equiv \exp[At]$:

$$v(t) = M(t)v(0) - \int_0^t ds\, M(t-s)[f(s) + \sqrt{2\kappa_e} f_{\rm in}(s)].$$

Supplementing the solution with an input-output relation for optics, $v_{\rm out} = \sqrt{2\kappa_e}v + f_{\rm in}$, and definition of the output quadratures [Eq. (2)]

$$V_{\rm out} = \mathcal{N}\int_0^\tau dt\, v_{\rm out}e^{-Gt}, \quad \mathcal{N} \equiv \sqrt{\frac{2G}{1-e^{-2G\tau}}},$$

allows one to write the solution for the latter as well:

$$V_{\rm out} = \mathcal{N}\int_0^\tau dt e^{-Gt}\bigg[f_{\rm in}(t) + \sqrt{2\kappa_e}\times\bigg(M(t)v(0) - \int_0^t ds M(t-s)[f(s) + \sqrt{2\kappa_e}f_{\rm in}(s)]\bigg)\bigg]. \tag{A2}$$

In order to quantify the impact of the different sources on the output optical state one can compute its covariance matrix (CM) with elements defined as

$$\mathbb{U}_{A_{\rm out}ij} \equiv \langle\{V_{{\rm out}\,i}, V_{{\rm out}\,j}\}\rangle, \quad i,j = 1,2, \tag{A3}$$

where $\{a,b\} = \frac{1}{2}(ab+ba)$. This CM appears to be a linear combination of CMs of the initial states and input fields.

To calculate $\mathbb{U}_{A_{\rm out}}$ we substitute the solution (A2) into the definition (A3), and interchange the order of integration and expectation. Assuming all the noises to be Markovian, namely

$$\begin{aligned} &\langle\{v(0),v(0)\}\rangle = \mathbb{U}_{A(0)} \oplus \mathbb{U}_{B_{\rm in}}, \\ &\langle\{f(t),f(t')\}\rangle = 2\kappa_i\mathbb{U}_{A_0} \oplus \gamma\mathbb{U}_{B_m}\cdot\delta(t-t'), \\ &\langle\{f_{\rm in}(t),f_{\rm in}(t')\}\rangle = \mathbb{U}_{A_{\rm in}} \oplus 0_{2\times 2}\cdot\delta(t-t'), \end{aligned} \tag{A4}$$

allows us to write the expression for the CM of the output optical mode state in simple form:

$$\begin{aligned} \mathbb{U}_{A_{\rm out}} = {}& T_{B_{\rm in}}\mathbb{U}_{B_{\rm in}} + T_{A_{\rm in}}\mathbb{U}_{A_{\rm in}} + T_{A_0}\mathbb{U}_{A_0} \\ &+ T_m^{\rm in}\mathbb{U}_{B_m} + T_{A(0)}\mathbb{U}_{A(0)}. \end{aligned} \tag{A5}$$

Here and in Eqs. (A4) by $\mathbb{U}_X$ we denote the CM of the mode in the state with annihilation operator X to match the notations of Eq. (4) of the main text. B_m is the state of the mechanical bath and $A(0)$ is the initial state of the intracavity optical mode; $B_{\rm in}$, $A_{\rm in}$, and A_0 (defined in the main text) correspond respectively to initial mechanical, optical input, and optical vacuum states. $\mathbb{U}_{B_{\rm in}}$ represents the input of the converter and $T_{B_{\rm in}}$ is hence the transfer coefficient. All other terms represent contributions of the noise in the converter.

The coefficients describing transfer of the mechanical bath and optical input variances to the optical output mode equal

$$\begin{aligned} T_m^{\rm in} &= \mathcal{N}^2 \iint_0^\tau dt dt' e^{-G(t+t')} 2\kappa_e \\ &\quad \times \int_0^t ds \int_0^{t'} ds'\, m_c(t-s) m_c(t'-s') \delta(s-s'), \\ T_{A_{\rm in}} &= \mathcal{N}^2 \iint_0^\tau dt dt' e^{-G(t+t')} \\ &\quad \times \bigg[\delta(t-t') - 4\kappa_e \int_0^{t'} ds'\, m_1(t'-s')\delta(t-s') \\ &\quad + 4\kappa_e^2 \int_0^t ds \int_0^{t'} ds'\, m_1(t-s) m_1(t'-s')\delta(s-s') \bigg], \end{aligned}$$

where

$$m_1(t) = M_{1,1}(t), \quad m_c(t) = M_{1,3}(t).$$

The other coefficients are rather involved as well, so we do not present the explicit expressions for those here.

By making estimations using the parameters of the recent experiment in Ref. [39], we conclude that the coefficients $T_{B_{\rm in}}$, $T_{A_{\rm in}}$, and T_{A_0} coincide with their counterparts in Eq. (4) (respectively $T\eta_o$, T_L, and $1 - T\eta_o - T_L$) with accuracy up to G/κ. The in- and outcoupling transmissions $\eta_{i,o}$ are equal to the ratio of the decay rate of the cavity to the readout channel κ_e to the total decay rate κ: $\eta_i = \eta_o = \kappa_e/\kappa$.

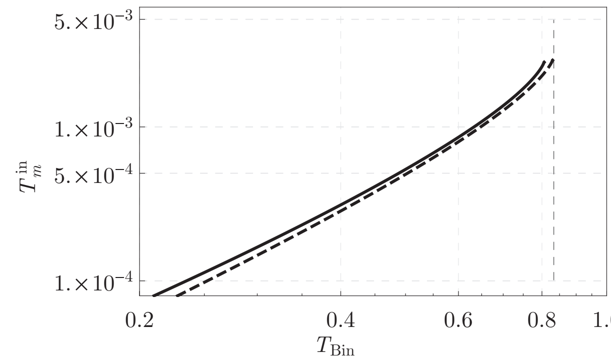

FIG. 7. Transfer of the mechanical bath variance $T_m^{\rm in}$ as a function of the signal transfer $T_{B_{\rm in}}$ calculated with parameters of the experiment reported in Ref. [39]. Solid and dashed lines represent respectively the full analytical solution (A5) and the approximate solution after adiabatic elimination of the cavity mode. The gray dashed line denotes the limit of possible transmittivity set by $\eta_o = \kappa_e/\kappa \approx 0.83$.

The coefficient $T_o^{\rm in}$ equals zero with same accuracy; hence, it is absent in Eq. (4). The coefficient $T_m^{\rm in}$ describes coupling to the noisy mechanical bath, so one should be cautious neglecting it. However, due to the small mechanical decoherence rate γ it is safe to omit it as long as the effective mechanical decoherence $\gamma n_{\rm th}$ ($n_{\rm th}$ is the mean mechanical bath occupation) is small: $\gamma n_{\rm th} \ll 1/\Delta\tau$. This condition is typically fulfilled in an experiment. We plot $T_m^{\rm in}$ as a function of $T_{B_{\rm in}}$ in Fig. 7. The figure proves that the impact of the mechanical bath is small enough to be neglected.

In summary, we verified that the effective model including the incoupling and outcoupling efficiencies (4) can be used to describe the high-quality conversion process that satisfies $\gamma n_{\rm th} \ll 1/\Delta\tau \ll G \ll \kappa$. These requirements, however, are anyway desirable if our conversion is considered to transfer highly nonclassical mechanical states to the traveling light beams.

[1] P. D. Drummond and M. Hillery, *The Quantum Theory of Nonlinear Optics* (Cambridge University Press, Cambridge, UK, 2014).
[2] *Quantum Squeezing*, edited by P. D. Drummond and Z. Ficek (Springer-Verlag, Berlin, 2004).
[3] S. Steinlechner, J. Bauchrowitz, M. Meinders, H. Müller-Ebhardt, K. Danzmann, and Roman Schnabel, Nat. Photonics **7**, 626 (2013).
[4] L. S. Madsen, V. C. Usenko, M. Lassen, R. Filip, and U. L. Andersen, Nat. Commun. **3**, 1083 (2012).
[5] S. Yokoyama, R. Ukai, S. C. Armstrong, Ch. Sornphiphatphong, T. Kaji, S. Suzuki, J. Yoshikawa, H. Yonezawa, N. C. Menicucci, and A. Furusawa, Nat. Photonics **7**, 982 (2013).
[6] A. Furusawa and P. van Loock, *Quantum Teleportation and Entanglement* (Wiley-VCH, Berlin, 2011).
[7] R. Filip, P. Marek, and U. L. Andersen, Phys. Rev. A **71**, 042308 (2005).
[8] R. Filip, Phys. Rev. A **69**, 052313 (2004).
[9] J. I. Yoshikawa, T. Hayashi, T. Akiyama, N. Takei, A. Huck, U. L. Andersen, and A. Furusawa, Phys. Rev. A **76**, 060301(R) (2007).
[10] J. I. Yoshikawa, Y. Miwa, A. Huck, U. L. Andersen, P. van Loock, and A. Furusawa, Phys. Rev. Lett. **101**, 250501 (2008).
[11] J. I. Yoshikawa, Y. Miwa, R. Filip, and A. Furusawa, Phys. Rev. A **83**, 052307 (2011).
[12] S. Yokoyama, R. Ukai, J. I. Yoshikawa, P. Marek, R. Filip, and A. Furusawa, Phys. Rev. A **90**, 012311 (2014).
[13] Y. Miwa, J. I. Yoshikawa, N. Iwata, M. Endo, P. Marek, R. Filip, P. van Loock, and Akira Furusawa, Phys. Rev. Lett. **113**, 013601 (2014).
[14] S. Lloyd and S. L. Braunstein, Phys. Rev. Lett. **82**, 1784 (1999).
[15] D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A **64**, 012310 (2001).
[16] S. D. Bartlett and B. C. Sanders, Phys. Rev. A **65**, 042304 (2002).
[17] S. Ghose and B. C. Sanders, J. Mod. Opt. **54**, 855 (2007).

[18] P. Marek, R. Filip, and A. Furusawa, Phys. Rev. A **84**, 053802 (2011).
[19] M. Yukawa, K. Miyata, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, Phys. Rev. A **88**, 053816 (2013).
[20] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014).
[21] M. Aspelmeyer, S. Gröblacher, K. Hammerer, and N. Kiesel, J. Opt. Soc. Am. B **27**, A189 (2010).
[22] *Cavity Optomechanics*, edited by M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt (Springer-Verlag, Berlin, 2014).
[23] S. Singh, G. A. Phelps, D. S. Goldbaum, E. M. Wright, and P. Meystre, Phys. Rev. Lett. **105**, 213602 (2010).
[24] O. Romero-Isart, M. L. Juan, R. Quidant, and J. I. Cirac, New J. Phys. **12**, 033015 (2010).
[25] O. Romero-Isart, A. C. Pflanzer, M. L. Juan, R. Quidant, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Phys. Rev. A **83**, 013803 (2011).
[26] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, Proc. Natl. Acad. Sci. USA **110**, 14180 (2013).
[27] W. Lechner, S. J. M. Habraken, N. Kiesel, M. Aspelmeyer, and P. Zoller, Phys. Rev. Lett. **110**, 143604 (2013).
[28] D. G. Grier, Nature (London) **424**, 810 (2003).
[29] *Optical Tweezers: Methods and Applications*, edited by M. J. Padgett, J. Molloy, and D. McGloin (Chapman and Hall, London, 2010).
[30] Y. Hayashi, S. Ashihara, T. Shimura, and K. Kuroda, Opt. Commun. **281**, 3792 (2008).
[31] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, Nature (London) **483**, 187 (2012).
[32] J. Gieseler, L. Novotny, and R. Quidant, Nat. Phys. **9**, 806 (2013).
[33] P. Jákl, A. V. Arzola, M. Šiler, L. Chvátal, K. Volke-Sepúlveda, and Pavel Zemánek, Opt. Express **22**, 29746 (2014).
[34] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. **75**, 281 (2003).
[35] S. Deléglise, I. Dotsenko, C. Sayrin, J. Bernu, M. Brune, J.-M. Raimond, and S. Haroche, Nature (London) **455**, 510 (2008).
[36] M. Yukawa, K. Miyata, T. Mizuta, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, Opt. Express **21**, 5529 (2013).
[37] G. Kirchmair, B. Vlastakis, Z. Leghtas, S. E. Nigg, H. Paik, E. Ginossar, M. Mirrahimi, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **495**, 205 (2013).
[38] T. A. Palomaki, J. W. Harlow, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Nature (London) **495**, 210 (2013).
[39] T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Science **342**, 710 (2013).
[40] S. G. Hofer, W. Wieczorek, M. Aspelmeyer, and K. Hammerer, Phys. Rev. A **84**, 052327 (2011).
[41] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, C. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, Proc. Natl. Acad. Sci. USA **108**, 16182 (2011).
[42] U. Leonhardt, *Measuring the Quantum State of Light* (Cambridge University Press, Cambridge, UK, 1997).
[43] M. Mehmet, S. Ast, T. Eberle, S. Steinlechner, H. Vahlbruch, and R. Schnabel, Opt. Express **19**, 25763 (2011).
[44] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[45] S. Takeda, T. Mizuta, M. Fuwa, P. van Loock, and A. Furusawa, Nature (London) **500**, 315 (2013).
[46] U. L. Andersen and R. Filip, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 2009), Vol. 53, p. 365, and references therein.
[47] J. Niset, J. Fiurášek, and N. J. Cerf, Phys. Rev. Lett. **102**, 120501 (2009).
[48] R. L. Hudson, Rep. Math. Phys. **6**, 249 (1974).
[49] R. Filip and L. Mišta, Jr., Phys. Rev. Lett. **106**, 200401 (2011).
[50] M. Wallquist, K. Hammerer, P. Rabl, M. Lukin, and P. Zoller, Phys. Scr. **2009**, 014001 (2009).
[51] K. Hammerer, A. S. Sorensen, and E. S. Polzik, Rev. Mod. Phys. **82**, 1041 (2010).